RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2025

FIRST YEAR (BATCH 2024-28)

MATHEMATICS

Date : 21/05/2025

Time : 11.00 am – 1.00 pm

Paper : 2MTMCOC1

Full Marks : 50

[Use a separate Answer Book for **each group**]

All the notations have their usual meaning.

Group - A

Answer **any two** questions from question nos. 1 to 3.

1. a) Let $\{x_n\}$ be a sequence of real numbers such that $\lim_{n\to\infty} x_n = l$. Then show that $\lim_{n\to\infty} \frac{x_1 + x_2 + ... + x_n}{n} = l.$ Is the converse of the above result true?

   b) Use the above result to prove that

   $\lim_{n\to\infty} \frac{1+\sqrt{2}+\sqrt[3]{3}+\sqrt[4]{4}+......+\sqrt[n]{n}}{n} = 1.$ [(3.5+1)+2]

2. a) Check the convergence of the series

   $\frac{1}{1\times 2^2}+\frac{1}{2\times 3^2}+.......+\frac{1}{n(n+1)^2}+.....$

   b) State D'Alembert's ratio test. Using D'Alembert's ratio test or otherwise test the convergence of the series $\sum_{n=1}^{\infty}\frac{3n-1}{n!}$. [3+(1+2.5)]

3. Let $x_1 = 0$ and $x_{n+1} = \sqrt{2+x_n}$, $n \in \mathbb{N}$

   a) Show that $\{x_n\}$ is bounded.

   b) Is $\{x_n\}$ convergent? If yes, find its limit. Else justify why it is not convergent. [2+4.5]

Group - B

Answer **all** the questions. maximum you can score is 12.

4. Let $D = \{2^n : n \in \mathbb{Z}\}$ where $\mathbb{Z}$ is the set of all integers. Prove that D is an abelian group with respect to multiplication. [5]

5. Let H & K be two subgroups of a group G. Prove that HK is a subgroup of G if and only if HK = KH. [5]

6. Define the order of an element of a group. If G is a group, $a \in G$ and $o(a) = 15$ find $o(a^6)$. [1+2]

7. Let G be a cyclic group of order 6. Find the number of generators of G. [2]

Group - C

Answer **any five** questions: [5 × 5]

8. Solve: $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$ [5]

9. Solve : $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$

10. Reduce the equation $x^3p^2+x^2yp+a^3=0$ to Clairaut's form by the substitutions $y=u$ and $x=\frac{1}{v}$ and obtain its complete primitive and singular solution.

11. Solve the differential equation $\frac{d^2y}{dx^2}+a^2y=\sec ax$ with the symbolic operator D.

12. Solve by the method of undetermined coefficients : $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x$.

13. Find the general solution of $(1+x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=(1+x)^2$ by the method of variation of parameters, it is given that y = x and y = e^{-x} are two linearly independent solutions of the corresponding homogenous equation.

14. Solve : $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=2x^2$.

15. Find the orthogonal trajectories of the system of co-axial circles $x^2+y^2+2\rho x+c=0$ where ρ is a parameter and c is a given constant.

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